

(3 Hours)

[Total Marks : 80]

N.B.

- 1) Q.1 is compulsory.
- 2) Solve any 3 questions out of remaining 5 questions.
- 3) Assumptions made should be clearly stated.
- 4) Draw the figures wherever required.

Q.1 Solve any four of the following questions.

- a) Prove using Mathematical Induction that n^3+2n is divisible by 3 for all $n \geq 1$ 5
- b) Explain the following terms with suitable example: 5
 - i) Partition set
 - ii) Power set.
- c) State the Pigeonhole principle and show that if any five numbers from 1 to 8 are chosen, then two of them will add to 9. 5
- d) Consider the function $f(x) = 2x-3$. Find a formula for the composition functions 5
 - i) $f^2 = f \circ f$
 - ii) $f^3 = f \circ f \circ f$
- e) Explain the bipartite graph with suitable example. 5

Q.2

- a) What is a transitive closure? Find the transitive closure of R using Warshall's algorithm where $A = \{1, 2, 3, 4, 5\}$ & $R = \{(x,y) \mid x-y = \pm 1\}$ 10
- b) What is a ring? Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Determine whether a set A with addition modulo 8 & multiplication modulo 8 is a commutative ring? Justify your answer. 10

Q.3

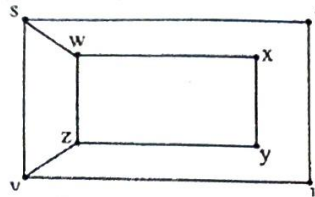
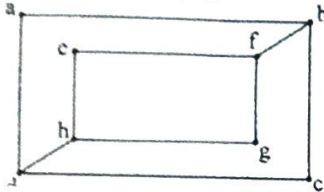
- a) A survey in 1986 asked households whether they had a VCR, a CD player or cable TV. 40 had a VCR. 60 had a CD player; and 50 had cable TV. 25 owned VCR and CD player. 30 owned a CD player and had cable TV. 35 owned a VCR and had cable TV. 10 households had all three. How many households had at least one of the three? How many of them had only CD player? 8
- b) Find the complete solution of a recurrence relation 6

$$a_n + 2a_{n-1} = n + 3 \text{ for } n \geq 1 \text{ and with } a_0 = 3$$
- c) Obtain CNF & DNF for the following expression: 6

$$p \leftrightarrow (\sim p \vee \sim q)$$

Q.4

- a) What is a group? Let $A = \{3, 6, 9, 12\}$ 10
 i) Prepare the composition table w.r.t. the operation of multiplication modulo 15.
 ii) Whether it is an abelian group? Justify your answer.
 iii) Find the inverses of all the elements.
 iv) Whether it is a cyclic group?
- b) What are the isomorphic graphs? Determine whether following graphs are isomorphic. 10



Q.5

- a) Let $X = \{1, 2, 3, 6, 24, 36\}$ & $R = \{(x,y) \in R \mid x \text{ divides } y\}$
 i) Write the pairs in a relation set R.
 ii) Construct the Hasse diagram.
 iii) What are the Maximal and Minimal elements?
 iv) Mention Chains and Ant chains from above set.
 v) Is this poset a lattice?

b) Define the term bijective function.

Let $f: \mathbb{R} \rightarrow \mathbb{R} \rightarrow \mathbb{R} - \left(\frac{2}{5}\right)$ be defined by $f(x) = \frac{2x-3}{5x-7}$.

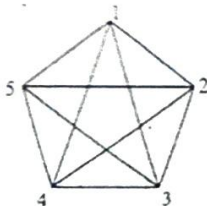
Whether a function is bijective? Justify your answer.

- c) Define minimum hamming distance. Consider $e: B^3 \rightarrow B^6$. Find the code words generated the parity check matrix H given below.

$$H = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Q.6

- a) Define with example Euler path, Euler circuit, Hamiltonian path, and Hamiltonian circuit. Determine if the following diagram has Euler circuit and Hamiltonian circuit. Mention the path/circuit. 6



- b) Let p denote the statement 'The food is good',
 q denote the statement 'The service is good' &
 r denote the statement 'The rating is 3 star.'

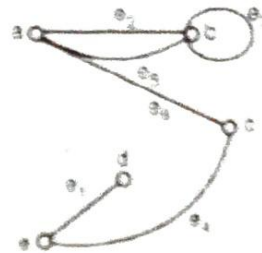
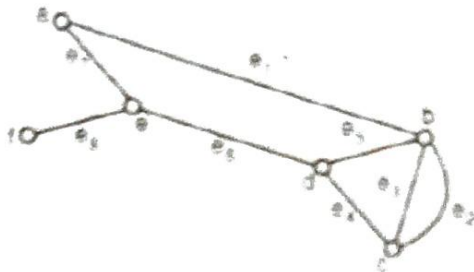
Write the following statements in a symbolic form-

- i) Either food is good or service is good or both.
- ii) The food is good but service is not good.
- iii) If both food & service are good then the rating is 3 star.
- iv) It is not true that a 3 star rating always means good food & good service.

8

- c) Find out the incidence matrix of following graphs.

6



Correction in Q. P. Code:11566
Discrete Structures & Graph Theory

Q.1 d)

Consider the function $f(x) = 2x-3$. Find answer for the composition functions

i) $f^2 = f \circ f$

ii) $f^3 = f \circ f \circ f$

Q.3 a)


A survey in 1986 asked households whether they had a VCR, a CD player or cable TV. **140** had a VCR. 60 had a CD player; and 50 had cable TV. 25 owned VCR and CD player. 30 owned a CD player and had cable TV. 35 owned a VCR and had cable TV. 10 households had all three. How many households had at least one of the three? How many of them had only CD player?
Here instead of 40 had a VCR consider 140 had a VCR

Q. 5 b)

Let $f: \mathbb{R} \rightarrow (7/5) \rightarrow \mathbb{R} - \left(\frac{2}{5}\right)$ be defined by $f(x) = \frac{2x-3}{5x-7}$.

Prove that it is a bijection. Hence find f^{-1} .

Here instead of $\mathbb{R} \rightarrow (7/5)$ consider $\mathbb{R} - (7/5)$


23/11/2022

(3 Hours)

Total Marks: 80

- N.B:** (1) Question No. 1 is compulsory.
 (2) Attempt any three questions out of the remaining five questions.
 (3) Figures to the right indicate full marks.
 (4) Make suitable assumptions wherever necessary.

- Q.1 (a) Compare linear and non-linear data structures. [05]
 (b) Explain the advantage of circular queue over linear queue. Write a function in C language to insert an element in circular queue. [05]
 (c) Define binary search tree. Discuss the case of deletion of a node in binary search tree if node has both the children. [05]
 (d) Write a C function to search a node in doubly linked-list. [05]

- Q.2 (a) Construct AVL tree for the following sequence: [10]
 67,34,90,22,45,11,2,78,37,122
 (b) Write algorithm for postfix evaluation. Demonstrate the same step by step for the expression: $9\ 6\ 7\ * \ 2\ / \ -$ [10]

- Q.3 (a) Write a program to perform following operations on a circular linked list: [10]
 i) insert a node from the end of the list, ii) delete first node,
 iii) count the number of nodes with even values, iv) display the list.
 (b) Write a C program to simulate linear queue as linked list. [10]

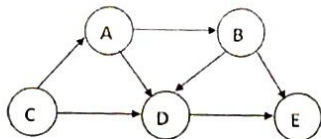
- Q.4 (a) Construct Huffman tree and find the Huffman codes for each symbol given below with frequency of occurrence: [10]

Symbol	p	g	e	r	i
Frequency	20	17	33	25	40

- (b) Explain the various ways to represent graph in the memory with example. [05]
 (c) Construct binary search tree from given traversal sequences: [05]

In-order traversal	D	E	B	A	C	F	G	I	H	J
Pre-order traversal	F	E	D	C	B	A	G	H	I	J

- Q.5 (a) Apply linear probing to hash the following values in a hash table of size 11 and find the number of collisions: 67,44,90,12,83,52,23,87,79. [10]
 (b) Define topological sorting. Perform topological sorting for the following graph: [10]



- Q.6 (a) Construct a B tree of order 3 by inserting the following given elements as: [10]
 77,97,75,64,53,14,26,49,82,59.
 Show the B tree at each step of insertion.
 (b) Write a function in C for DFS traversal of graph. Explain DFS graph traversal with suitable example. [10]

(3 hours)

Total Marks: 80

- N.B. 1. Question No. 1 is compulsory
 2. Attempt any **three** questions from remaining five questions
 3. Assume suitable data if **necessary** and justify the assumptions
 4. Figures to the **right** indicate full marks

Q1	A	Convert	05
		i) 123 in to binary	
		ii) $(AB9)_{16}$ in to Decimal	
		iii) $(351)_8$ in to decimal	
		iv) 129 in to BCD	
		v) 64 in to gray code	
	B	Draw the single and double precision format for representing floating point number using IEEE 754 standards and explain the various fields	05
Q1	C	Explain SR Flip Flop	05
	D	Differentiate between Hardwired control unit and Micro programmed control unit	05
Q2	A	Draw the flow chart of Booths algorithm for signed multiplication and Perform 5×2 using booths algorithm	10
	B	Explain the different addressing modes.	10
Q3	A	For 132.65 obtain the IEEE 754 standards of Single precision and Double precision format	10
	B	Explain Micro instruction format and write a microprogram for the instruction $ADD R_1, R_2$	10
Q4	A	Consider a 4-way set associative mapped cache with block size 4 KB. The size of the main memory is 16 GB and there are 10 bits in the tag. Find- 1. Size of cache memory 2. Tag directory size	10
	B	Explain Flynn's classification	10
Q5	A	Explain different types Distributed and Centralized bus arbitration methods	10
	B	Describe the detailed Von-Neumann Model with a neat block diagram	05
	C	Describe the characteristics of Memory.	05
Q6		Write Short notes on	20
	a)	Grey code, BCD, Excess-3 Code with example	
	b)	Encoder and Decoder	
	c)	Cache coherence	
	d)	Instruction Pipelining	

302, 303, 304, 305.

1

Correction in **Digital Logic & Computer Architecture**

Q.P Code: **14070**

Q.2 A) Draw the flow chart of Booths Algorithm for signed multiplication and perform

-5 x 2 using booths algorithm

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302, 303, 304, 305.

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Correction in **Digital Logic & Computer Architecture**

Q.P Code: **14070**

Q.2 A) Draw the flow chart of Booths Algorithm for signed multiplication and perform

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(3 Hours)

Total Marks: 80

- N.B: 1. Question No. 1 is compulsory
2. Attempt any 3 from remaining questions
3. Assume any suitable data if necessary and justify the assumptions

- Q.1 Attempt any **Four**. 20
- a) Give difference between random scan display and raster scan display.
 - b) Define Aliasing, Describe different antialiasing techniques.
 - c) Compare DDA and BRESENHAM line drawing algorithm.
 - d) Explain point clipping algorithm..
 - e) Give fractal dimension for KOCH curve.
- Q.2 a) Derive formula for mid-point circle algorithm. 10
- b) Given a line AB where A(3,1) and B(0,0) calculate all the points of line AB using DDA algorithm. 10
- Q.3 a) With neat diagram explain Composite transformation. 10
- b) Describe what is Homogeneous coordinates. 10
- Q.4 a) With neat diagram explain window to viewport coordinate transformation. 10
- b) With neat diagram explain Sutherland Hodgman polygon clipping algorithm. 10
- Q.5 a) Define projection, with neat diagram describe planar geometric projection. 10
- b) Describe properties of BEZIER curve. 10
- Q.6 a) Describe various principles of traditional animation. 10
- b) Write short note on Depth buffer algorithm. 10

Max. Marks : 80
 (Time: 03 hours)

- N.B. : (1) Question No.1 is **compulsory**
 (2) Attempt **any three** questions from Q.2 to Q.6
 (3) Figures to the right indicate full marks

Q. 1 (a) Find the Laplace Transform of $e^{2t} + 4t^3 - \sin 2t \cos 3t$ 05

(b) Find the Fourier series of $f(x) = x, \quad -\pi < x < \pi$ 05

(c) Calculate Spearman's coefficient of rank correlation from the following data 05

X:	12	17	22	27	32
Y:	113	119	117	115	121

(d) Find the constants a, b, c, d, e 05

if $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic

Q.2 (a) Determine whether the function $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{y}{x}$ is analytic and if so, find its derivative. 06

(b) A random variable X has the following probability distribution 06

X	0	1	2	3	4	5	6
P(X=x)	k	3k	5k	7k	9k	11k	13k

Find (i) k, (ii) $P(X < 4)$ (iii) $P(3 < X \leq 6)$

(c) Evaluate $\int_0^\infty e^{-2t} t \cos t dt$ 08

Q.3 (a) Find the Fourier series of $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}, \quad -\pi < x < \pi$ 06

(b) A continuous random variable has probability density function $f(x) = k(x - x^2); \quad 0 \leq x \leq 1$ 06
 Find (i) k, (ii) mean, (iii) variance

(c) Find the inverse Laplace transform of $\frac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$ 08

Q.4(a) Find the Laplace Transform of $f(t)$, 06
 where $f(t) = \cos t$, for $0 < t < \pi$ and $f(t) = \sin t$, for $t > \pi$

(b) Calculate the Karl Pearson's coefficient of correlation from the following data 06

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

(c) Find the Fourier series of $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 2\pi - x, & \pi \leq x \leq 2\pi \end{cases}$ 08

Q.5 (a) Find the inverse Laplace transform of $\frac{s}{(2s+1)^2}$ 06

(b) Find the Laplace transform of $t \left(\frac{\sin t}{e^t}\right)^2$ 06

(c) Find the lines of regression for the following data

X:	78	36	98	25	75	82	90	62	65	39
Y:	84	51	91	60	68	62	86	58	53	47

08

Q.6 (a) Find the mean and the variance of the following distribution

X	1	3	4	5
P(X=x)	0.4	0.1	0.2	0.3

06

(b) Find the inverse Laplace transform of $\log \left(1 + \frac{a^2}{s^2}\right)$ 06

(c) Find the analytic function $f(z) = u + iv$ whose imaginary part is $v = x^2 - y^2 + \frac{x}{x^2 + y^2}$ 08

Time: 3 Hours

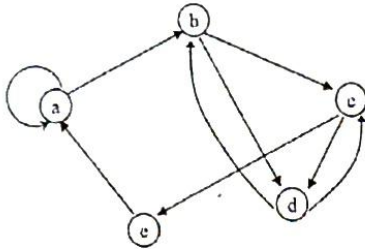
Marks: 80

N.B. : (1) Question Number 1 is compulsory

- (2) Solve any three questions from the remaining questions
- (3) Make suitable assumptions if needed
- (4) Assume appropriate data whenever required. State all assumptions clearly.

Q.1 Solve any four of the following questions.

- a) Prove using Mathematical Induction that $1+5+9+ \dots + (4n-3) = n(2n-1)$ 5
- b) Find the relation set & relation matrix for the following digraph. Determine in degree & out degree of each vertex. 5



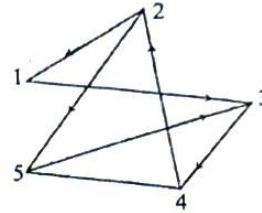
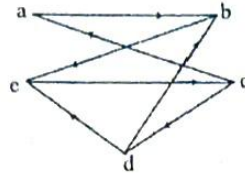
- c) State the pigeon hole principle. If 30 people are assembled in a room, then show that of them must have their birthday on the same day of a week. 5
- d) Explain the following terms with suitable example: 5
 - i) Eulerian graph
 - ii) Quantifier
- e) What is a partial order relation? Determine the hasse diagram for following relation 5
 $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (3, 4), (4, 4)\}$

Q.2

- a) What is a transitive closure? Let $A = \{a_1, a_2, a_3, a_4, a_5\}$. Find the transitive closure of R using Warshall's algorithm where relation matrix M_R is given as follows- 10

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

- b) What are the isomorphic graphs? Determine whether following graphs are isomorphic.



Q.3

- a) Among the integers 1 to 300,
 i) How many of them are not divisible by 3 nor by 5 nor by 7?
 ii) How many of them are divisible only by 3?
- b) There are 6 Communication Skills books, 8 Engg. Mathematics books, 10 books on Programming. How many ways can be used to choose 2 books of different categories from them?
- c) What is a partition set? Determine whether each of the following is a partition. Justify your answer.

Let $X = \{1, 2, 3, \dots, 8, 9\}$. Determine whether or not each of following is a partition

- (a) $\{\{1, 3, 6\}, \{2, 8\}, \{5, 7, 9\}\}$
 (b) $\{\{2, 4, 5, 8\}, \{1, 9\}, \{3, 6, 7\}\}$
 (c) $\{\{1, 5, 7\}, \{2, 4, 8, 9\}, \{3, 5, 6\}\}$
 (d) $\{\{1, 2, 7\}, \{3, 5\}, \{4, 6, 8, 9\}, \{3, 5\}\}$

Q.4

- a) What is a group? Let $A = \{5, 10, 15, 20\}$
 i) Prepare the composition table w.r.t. the operation of multiplication modulo 25.
 ii) Whether it is an abelian group? Justify your answer.
 iii) Find the inverses of all the elements.
 iv) Whether it is a cyclic group?
- b) What is a ring? Let $A = \{0, 1, 2, 3, 4, 5\}$. Determine whether a set A with addition modulo 6 & multiplication modulo 6 is a commutative ring? Justify your answer.

Q.5

- a) Define a lattice. Prove that in a distributive lattice the complement of any element is unique. Determine whether D_{105} is a distributive lattice. Find the complements of all elements.
- b) Define the term bijective function. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = 2x$. Determine whether it is a bijective function.

c)

6

Draw the graph G corresponding to each adjacency matrix

$$(a) A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$(b) A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

Q.6

a)

8

Let $A = \{2, 5, 9, 13, 16\}$

$R = \{(2, 5), (2, 13), (16, 5), (16, 13), (9, 13), (5, 16)\}$

$S = \{(2, 9), (2, 13), (5, 13), (9, 16), (5, 16)\}$

Compute (i) S^{-1} (ii) $(R \cup S) \cap S^{-1}$ (iii) $\bar{R} \cap S$ (iv) \bar{R}

b) What is a planer graph? A connected planer graph has 8 vertices having degrees 2,2,2,3,3,3,4,4. How many edges are there in this graph? 6

c) Write the following statements in a symbolic form using quantifiers. Assume a suitable data wherever applicable. 6

- i) All students have taken a course in mathematics.
- ii) There is a girl student in a class who is also a sports person.
- iii) Some students are intelligent, but not hardworking.

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Correction in Q.P.Code: 10021108 Discrete Structures & Graph Theory

Q.4

- a) What is a group? Let $A = \{5, 10, 15, 20\}$ 10
i) Prepare the composition table w.r.t. the operation of addition modulo 25.
ii) Whether it is an abelian group?
iii) Find the inverses of all the elements, if exist?
iv) Whether it is a cyclic group?

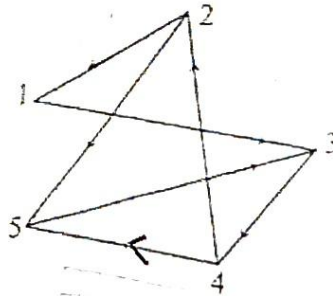
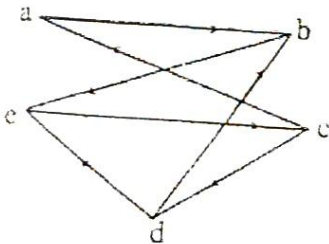
Instead of Multiplication it is addition modulo 25
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2) B)

- b) What are the isomorphic graphs? Determine whether following graphs are isomorphic. 1



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(Time: 3 Hours)

Total Marks: 80

- N.B. 1. Question No. 1 is compulsory
 2. Attempt any **three** questions from remaining five questions
 3. Assume suitable data if **necessary** and justify the assumptions
 4. Figures to the **right** indicate full marks

Q1	A	Define the terms Computer Organization and Computer Architecture and differentiate between them with an example.	05
Q1	B	Explain IEEE 754 Floating point representations.	05
	C	Define Instruction cycle. Explain it with a detailed state diagram.	05
	D	How Hardwired control unit differs from Micro programmed control unit	05
Q2	A	Draw a neat flow chart of Booths algorithm for signed multiplication and Perform 7×-3 using booths algorithm	10
	B	Explain the different addressing modes.	10
Q3	A	Explain state table method of designing a Hardwired Control unit	10
	B	Represent 3.5 in IEEE 754 Single precision Format	05
	C	Explain SR Flip Flop	05
Q4	A	Consider a 4-way set associative mapped cache with block size 4 KB. The size of the main memory is 16 GB and there are 10 bits in the tag. Find- 1. Size of cache memory 2. Tag directory size	10
	B	Explain Micro instruction format and write a microprogram for the instruction $ADD R_1, R_2$	10
Q5	A	A program having 10 instructions (without Branch and Call instructions) is executed on non-pipeline and pipeline processors. All instructions are of same length and having 4 pipeline stages and time required to each stage is 1nsec. (Assume the four stages as Fetch Instruction, Decode Instruction, Execute Instruction, Write Output) i.) Calculate time required to execute the program on Non-pipeline and Pipeline processor. ii) Show the pipeline processor with a diagram.	10
	B	Write a short note on cache coherency.	05
	C	Describe the characteristics of Memory.	05
Q6	A	Explain Flynn's classification.	10
	B	Explain different types Distributed and Centralized bus arbitration methods	10

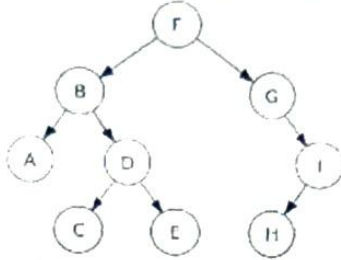
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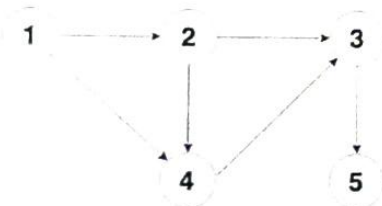
N.B: (1) Question No. 1 is compulsory

(2) Attempt any three questions out of the remaining five questions

- Q.1 (a) Define ADT. Write ADT for Queue data structure. [05]
(b) Find the in-order, pre-order, post-order traversal [05]



- (c) Differentiate between Linked list and Array [05]
(d) Explain application of Binary tree [05]
- Q.2 (a) Apply Huffman coding for following examples. Determine the code for the following characters. "CONSTRUCTION" [10]
(b) Consider a hash table with size = 10. Using Linear probing, insert the keys 28, 55, 71, 67, 11, 10, 90, 44 into the table. [10]
- Q.3 (a) Write an C program to check the well-formedness of parenthesis in an algebraic expression using the Stack data structure. [10]
(b) Construct AVL for the given elements 27,25,23,29,35,33,34 [10]
- Q.4 (a) Write a program to perform the following operations on the Doubly linked list: [10]
i. Insert a node at the end
ii. Delete a node from the beginning
iii. Search for a given element in the list
iv. Display the list
(b) Write DFS algorithm. Show DFS traversal for the following graph with all the steps. [10]



- Q.5** (a) Define Data Structure. Explain its type with an example [10]
(c) Explain B tree. Draw the B-tree of order 3 created by inserting the following data arriving in sequence: 50, 25, 10, 5, 7, 3, 30, 20, 8, 15 [10]
- Q.6** (a) Draw the Stack structure in each case when the following operations are performed on an empty stack. [10]
i. PUSH A, B, C, D, E, F
ii. POP two letters
iii. PUSH G
iv. POP H
v. POP four letters
vi. PUSH I, J
vii. POP one letter
- (b) Write a C program for polynomial addition using a Linked-list. [10]

(Time: 3 Hours)

Marks: 80

- N.B: 1) Question **number 1** is compulsory.
2) Attempt **any three** out of the remaining.
3) Assume suitable data if **necessary** and justify the assumptions.
4) Figures to the **right** indicate full marks.

Q 1

- A What is computer graphics and discuss its representative uses [5]
- B Explain traditional animation techniques [5]
- C Describe homogeneous coordinate system [5]
- D Explain point clipping method with suitable example [5]

Q 2

- A Given a triangle ABC with coordinates A (0, 0), B (10, 0), C(0,10). Apply following transformations in sequence [10]
 - i. Translate the triangle by translation parameters (20, 30) units.
 - ii. Rotate the triangle by 90° .Find the new coordinates of the triangle.
- B Explain Cohen Sutherland line clipping method with suitable example [10]

Q 3

- A Derive midpoint ellipse drawing algorithm with suitable diagrams [10]
- B Discuss principles of animation. [10]

Q 4

- A What is window and viewport. Derive the transformation matrix for a window-to-viewport transformation [10]
- B Explain and write matrices for 3D rotation about X, Y and Z axes. [10]

Q 5

- A What is aliasing effect? Explain antialiasing techniques [10]
- B Calculate all the points on the line from point A(0,0) to point B(8,10) using DDA line drawing method. [10]

Q 6

- A Derive the 2D transformation matrix for scaling with respect to fix point. [10]
- B Explain depth buffer method with suitable diagrams [10]

(Time: 3 hours)

Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks.

(a) Find Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$ given that $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-(1/4s)}$ [5]

(b) Calculate Spearman's rank correlation coefficient for the following data: [5]

X	32	55	49	60	43	37	43	49	10	20
Y	40	30	70	20	30	50	72	60	45	25

(c) Find inverse Laplace transform of $\frac{2s-1}{s^2+8s+29}$ [5]

(d) If $f(z) = qx^2y + 2x^2 + ry^3 - 2y^2 - i(px^3 - 4xy - 3xy^2)$ is analytic, find the values of p, q, and r [5]

Q2 (a) Find Laplace transform of $e^{3t} f(t)$ where $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$ [6]

(b) Two unbiased dice are thrown. If X represents sum of the numbers on the two dice. Write probability distribution of the random variable X and find mean, standard deviation, and $P(|X-7| \geq 3)$ [6]

(c) Obtain Fourier series for $f(x) = x \sin x$ in the interval $0 \leq x \leq 2\pi$. [8]

Q3 (a) Using Milne-Thompson's method construct an analytic function $f(z) = u + iv$ in terms of z where $u + v = e^x(\cos y + \sin y) + \frac{x-y}{x^2+y^2}$ [6]

(b) Using convolution theorem find the inverse Laplace transform of $\frac{(s+3)^2}{(s^2+6s+5)^2}$ [6]

(c) Fit a parabola $y = a + bx + cx^2$ to the following data and estimate y when $x=10$ [8]

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

Q4 (a) Find Laplace transform of $e^{-(1/2)t} t f(3t)$ if $L\{f(t)\} = \frac{1}{s\sqrt{s+1}}$ [6]

- (b) Find half range sine series for $f(x) = x - x^2$, $0 < x < 1$. [6]

Hence deduce that
$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots = \frac{\pi^3}{32}$$

- (c) Given regression lines $6y=5x+90$, $15x=8y+130$, $\sigma_x^2 = 16$. [8]
Find i) \bar{x} and \bar{y} , ii) r , iii) σ_y^2 and iv) angle between the regression lines

- Q5 (a) Can the function $u = r + \frac{a^2}{r} \cos\theta$ be considered as real or imaginary part of an analytic function? If yes, find the corresponding analytic function. [6]

- (b) An unbiased coin is tossed three times. If X denotes the absolute difference between the number of heads and the number of tails, find moment generating function of X and hence obtain the first moment about origin and the second moment about mean. [6]

- (c) Evaluate $\int_0^\infty e^{-2t} \cosht \int_0^t u^2 \sinh u \cosh u du dt$ [8]

- Q6 (a) Find inverse Laplace transform of $\frac{1}{(s-2)^4(s+3)}$ using method of partial fractions. [6]

- (b) If a continuous random variable X has the following probability density function [6]
$$f(x) = \begin{cases} k e^{-\frac{x}{4}}, & \text{for } x > 0 \\ 0, & \text{elsewhere} \end{cases}$$
 find k , mean and variance.

- (c) Find half range cosine series for $f(x) = x$, $0 < x < 2$. [8]

Hence deduce that i) $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \dots = \frac{\pi^4}{96}$

ii) $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots = \frac{\pi^4}{90}$
